##### A global inter-country economic model based on linked input-output models DRAFT

Robert G. Levy, Thomas P. Oléron Evans, Alan G. Wilson

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A global model is presented that can be used as the basis for assessing the impacts of future changes in trade, migration, security and development aid. The model is based on input-output models for 40 countries, linked with trade data at the sector level. This is made possible by the World Input-Output Database, a collection of input-output tables for 40 countries across 15 years, and by databases of commodities and services trade from the UN. The model is constructed using a minimum number of assumptions, and is based as far as possible on empirical observation. Some initial analysis of the model and its properties are also presented

## 1 Introduction

The objective of this paper is to present global economic model that can be used as the basis for assessing the impacts of future changes in trade, migration, security and development aid. The model presented here represents a first 'proof of concept' step towards this ambitious goal. The economies of individual countries are represented as 35-sector input-output models each of which is linked through trade flows representing imports and exports. This has recently been made feasible by the publication of the World Input-Output Database (WIOD) Timmer2012, a collection of input-output tables (IOTs) for 40 countries across 15 years, from 1995 to 2009. The IOTs are linked through data from the UN covering trade in both goods[[1]](#footnote-1) and services[[2]](#footnote-2).

The remainder of this paper is structured as follows: Section 2 gives an overview of existing work in this area. Section 3 gives a description of the present system and outlines how data is used to calibrate the parameters of the model. The algorithm used to calculate the output of the model is described in section 4 and some preliminary results are given in section ??.

Some concluding comments are added in section 6.

## 2 Existing Global Economic Models

In the mid-1970s, the creator of input-output economics, Wassily Leontief, had just won the Nobel prize for Economics and took the prestige that this bestowed on him as an opportunity to announce a very ambitious project to model the global economy:

Major efforts are underway to construct a data base for a systematic input-output study not of a single national economy but of the world economy viewed as a system composed of many interrelated parts [...] Preliminary plans provide for a description of the world economy in terms of twenty-eight groups of countries, with about forty-five productive sectors for each group. [?]

Some 20 years later, Faye Duchin, a former research assistant of Leontief, described how Leontief's efforts in this area have largely been ignored by economists, describing the departures from the standard, neoclassical modelling in terms of price elasticity and elasticity of substitution as being "too great to ignore" Duchin2004. See section 3.1 for more on this subject.

In the years since Leontief published his global model, Input-output analysis has been largely restricted to regional studies, of which Akita1993, Khan1999 and Luo2013a are examples; and studies related to energy and the environment, such as Leontief1970, Joshi1999, Bergh2002 and Hendrickson2006.

However, much more recently, attention has returned to input-output modelling in a global context more generally. Tukker2013 describe how several multi-regional input-output (MRIO) models have been developed in the very recent literature. These are, along with the WIOD which is used in the present model, EORA Lenzen2012, EXIOPOL Tukker2013a and the slightly more mature, and proprietorial GTAP Walmsley2012.

## 3 The System Description

The model presented here has countries, the economies of which are divided into productive sectors. Although each sector produces many distinct goods, these goods are considered to be perfect substitutes, allowing the terms 'sector' and 'product' to be used interchangeably throughout this paper. All goods flows are given by value, measured in current price US$. For the remainder of this paper the terms 'quantity' and 'value' will be used interchangeably. Goods produced domestically are labelled with a dagger superscript () and imported goods with an asterisk superscript. Table 1 shows the quantities, taken from data published by the WIOD, which characterise a country's economy for a particular year. Note that for clarity, no time subscript is added. In future time-series analyses such a subscript would have to be added.

### 3.1 Input-Output Tables

Input-output is, at its heart, an accounting methodology. The products produced by and imported into a given country in a given year must be either: sold as inputs to other sectors, 'intermediate supply'; supplied to the 'final demand' of consumers and the government; invested; or exported. The total amount imported and produced must equal the amount used, consumed, invested and exported for each sector.

By simple summation, total production of sector in country can be defined as the sum of all intermediate supply, plus supply to final demand, investment and export:

(1)

where is the intermediate supply from domestic sector to sector in country , is final demand for domestic sector , is investment and are exports. The total import of sector in country is the sum of all intermediate supply by imported goods, plus demand for and investment of imported goods (in this iteration of the model, no imported goods are exported; that is, *re-exports* are set to zero):

(2)

By assembling these data values in a particular arrangement, an input-output table can be constructed, , as described by Miller1985. Neglecting the superscript for clarity, the input-output table (IOT) is defined as follows:

(3)

Table 1 shows a summary of the quantities used in (3). It will often be convenient to gather those quantities having a single subscript into vectors, and those with two subscripts into matrices.

|  |  |
| --- | --- |
|  | final demand on sector in country |
|  | investment of sector in country |
|  | export of sector in country |
|  | intermediate demand on domestic sector by sector in country |
|  | intermediate demand on import sector by sector in country |
|  | |

Table 1: Quantities from data which define a country's economy

We can then characterise a country's economy through the -vectors , and , and by the matrices and . In matrix form, may be written:

(4)

### 3.2 A Country Model

In the standard input-output model, each country is described by the input-output table described in section 3.1 above. From the elements of and , each sector has a complete 'recipe' for making its output, in terms of the quantities of each good used as input, both domestic and imported. For the remainder of this section, the country superscript will be excluded whenever its presence is clear from the context.

#### Technical Coefficients

By dividing intermediate flows by total output, we can arrive at a set of *technical coefficients* which define the input of one sector required per unit output of another. The amount of good required by sector to produce a single unit of output is thus:

(5)

for domestically produced , and

(6)

for imported . There are therefore technical coefficients for each country[[3]](#footnote-3). These technical coefficients then allow the intermediate requirements (both domestic and imported) to be calculated for any exogenously given vector of final, investment and export demands. For sector ::

or, in matrix representation:

(7)

Having calculated total domestic production, we can then calculate the import required to satisfy intermediate and final demands as:

(8)

Thus the domestic total production and the imports are completely determined from demand and the technical coefficients.

#### Import Ratios

Since the eventual goal of this model is to represent all the countries in the world, many of the country IOTs will have to be estimated from available data (in fact, all except the 40 of the WIOD will). It is therefore crucial that the model be as parsimonious as possible in terms of parameters. To this end, the model features two simplifications inspired by the description of Leontief's global model given by Duchin2004.

Leontief assumed that engineers in an importing country do not care where a product originated from; they will simply know that domestic production does not meet their demand, and instead demand a perfectly-substitutable imported good. In a similar spirit, when a product in the present model arrives at the shores of an importing country, it enters a national warehouse along with domestically produced goods, at which point the two become indistinguishable[[4]](#footnote-4). The only thing which is specified by the model is the fraction of all goods used domestically which must come from imports. This fraction remains fixed per country and per sector, and is called the *import ratio*. It is calculated as:

(9)

where , and are the total production, export and import of sector , calculated via equations (1) and (2) respectively. The term represents the goods used domestically. It includes intermediate flows required to fulfil export demand, as well as direct flows to final demand and investment, but excludes direct flows to export. This is because imports may not directly supply export demand as per equation (4).

The assumption of fixed import ratios makes easier the job of estimating countries for which the WIOD has no data, since it reduces the number of technical coefficients by half. This reduced set of technical coefficients means that only total inter-sector flows, , equivalent to in equation (4), need to be estimated, along with an -vector, , of import ratios. Following equation (5), the technical coefficients can then be calculated as

(10)

allowing the calculation of as per equation (7):

(11)

and hence of from equation (9):

(12)

where is the matrix of technical coefficients, is an matrix whose diagonal elements are the import ratios, , and is the identity matrix. Having calculated the total production, , in equation (11), the inter-sector flows, , can be recovered as follows:

(13)

(14)

Additional benefits to the import ratios assumption are that final demand and investment have only elements , rather than the elements shown in equation (3). In a 35 sector model, such as that used by the WIOD, the number of data points to be estimated in order to add a new country thus reduces from (37 being the 35 sector columns plus and ) to (the columns plus , and the import ratios).

### 3.3 An International Trade Model

In standard inter-regional input-output modelling (IRIO), each sector in each country is explicit about which countries it gets its imports from. This requires each sector to have technical coefficients, an onerous data requirement.

The WIOD present all these technical coefficients in the world input-output tables (WIOTs) which they publish, but in order to include countries for which the WIOD publishes no data, a second assumption must be made, related to what Leontief via Duchin2004 refers to as "export shares".

#### Import Propensities

The assumption is that a country gets each product from other countries in fixed proportions. Thus, country will always import the same proportion of total demand for product from country . We refer to these fixed proportions as *import propensities* as they describe a country's propensity to import from each other country. The propensity for country to import good from country is given by

(15)

where is the trade flow of good from country to country . The are taken from the UN COMTRADE database, with a mapping from 6-figure product code to sector kindly provided by the WIOD team.

Given the import requirements of each country from equation (12) and the import propensities from (15), the export demand on sector in country due to demand from country can be calculated as:

and total export demand in country is therefore:

(16)

Equations (9)–(16) thus describe a system which defines the total productions, , of all sectors in all countries; the intermediate input-output flows, and ; imports, ; exports, ; and all trade flows, , given a set of technical coefficients, ; import ratios, ; trade propensities, ; and final demands[[5]](#footnote-5), . For an exogenous change in any of the latter four parameters, all of the former six groups of flow can be found.

## 4 The Model Algorithm

As outlined in section 3 above, the model takes four groups of parameters and produces a complete set of flows within countries (input-output flows) and a complete set of trade flows (imports and exports) between countries. To recap, the four groups of parameters are:

1. Final demand, : the quantity of a good consumed by the public and the government of a particular country.

2. Import propensities, : the proportion of country 's total of import of good which comes from country .

3. Import ratios, : the proportion of country 's total demand for good which is supplied by imports.

4. Technical coefficients, : the quantity of good required in country to make a single unit of good .

Figure 1 shows a schematic version of the algorithm for calculating imports and exports from the four groups of parameters. Initially all export demands are set to 0. Inside each country, total demand is calculated using the technical coefficients and import ratios, as per equation (11). From this, import demand can be calculated via the import ratios as in equation (12). Turning to the international trade model, these import demands can be turned into export demands using the import propensities according to equation (16). At this point, each country model has a been given new set of export demands leading, again via the technical coefficients and the import ratios, to a new set of import demands.

An iteration condition is then checked: do exports and imports balance globally to within some small ? If they do balance, the iteration is complete. If not, we continue to iterate between the country-level and the international-level models until the system of imports and exports converges.

[>=latex, very thick] all nodes=[shape=rectangle] box=[draw] [box,label=above:Initialisation](init) ; [box, below right= and -.5cm of init, align=center, label=below:Country-Level Model] (country)

; [box, above right= and -1cm of country](trade) [label=above:International Trade Model] ; [box, right= of country, align=center](iteration) [label=60:Iteration Condition] Is stop condition met?

; [box, below=of iteration, align=center](result) [label=left:Results]

; [below left=0.5cm and -2cm of country, align=left] Exogenous:

, , , ; [->] (init) |- (country); [->] (country) |- (trade); [->] (trade) -| (iteration); [->] (iteration) – node[above]no (country); [->] (iteration) – node[right]yes(result);

Figure 1: The model algorithm: total production, imports and exports are calculated for a given set of exogenous parameters.

## 5 Analysis

A simple way to find the sectors most important to the global economy is to reduce by $1 final demand for each sector in turn, measuring the effect of the other countries and sectors in the model.

Figure 1 shows the 10 largest sectors in terms of their impact on the total output, , of every other sector in every other country. The vehicles sector in China is the most important sector by this measure, a $1 reduction in final demand in this sector leading to a $2.31 reduction in total output worldwide. Figure 1 shows the ten countries most affected by Chinese vehicles, as measured by the total impact on all sectors. China itself tops this table with a $9.55 reduction in total output due to a $1 reduction in output from the vehicles sector. In figure 1, the response of the Chinese economy is broken down by sector. The electricals sector is the most strongly affected, with a $1.62 reduction in total output. Also strongly affected are the vehicles sector itself ($1.53), metals ($1.44) and chemicals ($0.95).

It might be assumed from figure 1 that the electricals industry is heavily demanded by the vehicles industry, but the relevant technical coefficient is only 0.06. It hence takes just 6 of electricals to make a dollar's worth of vehicles, compared to almost 12 of metals. Furthermore, metals is more demanded by the Chinese economy as a whole, requiring $1.06 to make one dollar of every sector, against 77 of electricals.

### A random 'hopper'

Consider the problem of finding routes which products in the model might take to get from sector in country to sector in country . A random walker, or 'hopper', will starting in , and first decide whether to hop to another sector in or to hop abroad. The probability that the hopper hops abroad is given by the import propensity, . If hopping domestically, the hopper hops randomly to another sector, , with probability proportional to the technical coefficient . The coefficient is set to zero to avoid paths involving sector itself. If hopping abroad, the hopper arrives in country with a probability proportional to the import propensity .

The hopper hops in this manner for some fixed, arbitrary number of hops. If the hopper arrives at the destination sector, in country within this number of hops, the route taken is recorded. When either the destination is reached or the maximum number of hops is exceeded, the hopper starts again from sector in country .

[response summed across all countries/sectors]

|  |  |  |
| --- | --- | --- |
| Sector | Country |  |
| Vehicles | China | -2.31 |
| Plastics | China | -2.30 |
| Metals | Korea | -2.25 |
| Vehicles | Korea | -2.25 |
| Textiles | China | -2.23 |
| Construction | China | -2.16 |
| Utilities | Korea | -2.14 |
| Leather | China | -2.14 |
| Wood | China | -2.10 |
| Metals | China | -2.05 |
|  | |

[response to vehicles sector in China]

|  |  |
| --- | --- |
| Country |  |
| China | -9.56 |
| Japan | -8.73 |
| USA | -6.95 |
| Italy | -6.38 |
| Germany | -5.91 |
| Poland | -4.91 |
| Czech Rep. | -4.55 |
| Korea | -4.31 |
| France | -4.22 |
| Brazil | -4.18 |
|  | |

[response of Chinese sectors to vehicles sector in China]

|  |  |
| --- | --- |
| sector |  |
| Electricals | -1.62 |
| Vehicles | -1.53 |
| Metals | -1.44 |
| Chemicals | -0.95 |
| Machinery | -0.54 |
| Mining | -0.42 |
| Plastics | -0.39 |
| Utilities | -0.35 |
| Textiles | -0.23 |
| Wholesale Trade | -0.22 |
|  | |

Table 2: Response () to a $1 reduction in final demand for the shown sector, in terms of difference in the total output, , of other sectors. All tables show the largest 10 only.

### A unified network approach

The flow of goods between countries can be viewed as a weighted, directed network, where countries are nodes, and the weights of each edge represents the magnitude of the flow between them Nystuen1961,Serrano2003,Bhattacharya2008,Baskaran2011. Similarly with the sector-sector flows which constitute an input-output model Blochl2011. Network science has developed useful ways to analyse the sort of weighted, directed networks which the constitute the present model, but a single network representation is required which combines the international and the sub-national networks.

Recall from section 3.2 above, that goods arriving at the shores of an importing country are put into a warehouse along with domestically produced goods, at which point the two become indistinguishable. If an additional assumption is made that domestic sectors take from this warehouse by method of a random sample, then the fraction of goods in each sample from abroad will be identical, as will the fraction of imported goods from each exporter. These fractions will be set by the import ratios and import propensities respectively.

This additional assumption allows us to specify a complete system of intermediate (input-output) flows, , from sector in country to sector in country :

(17)

Equation (20) can be understood as follows: sector in country requires an amount of sector 's good to produce its total output; a fraction of this will be supplied by imports; a fraction of these imports will come from country . Note that since countries do not import their own exports, , and the expression holds trivially for .

The fractions of each type of demand for in country which was exported by country are given by similar expressions:

(18)

(19)

(20)

where, as previously, is final demand, is export demand and are investments.

Along with the sector-to-sector flows given by equation (14), this allows the representation of all the flows in the present model as a single network, and standard network analysis techniques can then be used.

We can visualise the changes which occur following the reduction of demand in China for the Vehicles sector by $1. The change to every flow (both and type flows) is recorded and visualised in figure 2.

Figure 2: A network representation of the top six most-affected countries following a reduction in final demand for the Chinese vehicles sector. Node size is proportional to eigenvector centrality, and edge width is proportional to the change in flow.

Because of the highly aggregated nature of the WIOD sectors (over 6,000 product categories aggregated into 35 sectors) the highest technical coefficients tend to be those relating to within sector flows, . This results in the vast majority of international trade taking place within sectors, e.g. from sector in country to sector in country . The inter-sector dependencies are thus swamped by the intra-sector flows. To study inter-sector flows, the intra-sector flows, , can be set to zero before any network analysis is performed.

## 6 Conclusions

1. comtrade.un.org/db [↑](#footnote-ref-1)
2. unstats.un.org/unsd/servicetrade/ [↑](#footnote-ref-2)
3. Recall that single year is assumed throughout this treatment. In time series analyses there will be technical coefficients for every country in every year. [↑](#footnote-ref-3)
4. Note that this concept is referred to in Miller1985 as *import similarity* [↑](#footnote-ref-4)
5. In the first iteration of the model, investments, , are set to zero. Data is provided by the WIOD for investments but it is not used in this iteration of the model. [↑](#footnote-ref-5)